Energy Efficient Design for MIMO Two-Way AF Multiple Relay Networks

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Abstract—This paper studies the energy efficient transmission and the power allocation problem for multiple two-way relay networks equipped with multi-input multi-output antennas where each relay employs an amplify-and-forward strategy. The goal is to minimize the total power consumption without degrading the quality of service of the terminals. In our analysis, we start by deriving closed-form expressions of the optimal powers allocated to the terminals. We then employ a strong optimization tool based on the particle swarm optimization technique to find the optimal power allocated at each relay antenna. Our numerical results illustrate the performance of the proposed scheme and show that it achieves a sub-optimal solution very close to the optimal one.

Index Terms—Two-way relay networks, amplify-and-forward, particle swarm optimization.

I. INTRODUCTION

Energy efficient techniques have attracted enormous attentions from the green communication research community. Achieving energy saving while respecting a certain pre-defined data rate becomes one of the most important considerations for wireless networks. Two-Way Relay Networks (TWRNs) have been recently considered as a promising energy saving technique by reducing the transmission powers of the network [1], [2]. By using this technique, data exchange between terminals via relays can be accomplished within two transmission phases only. During the first phase, the terminals transmit their signals to the relays simultaneously. Then, in the second phase, the relays broadcast their signals to the terminals. Finally, the terminals apply a self-interference cancelation operation to extract the desired data [3], [4].

Since the total power in a relay system may be limited, it is important to distribute the power budget in an efficient way. Furthermore, several studies have discussed the energy efficiency of TWRNs with Amplify-and-Forward (AF) strategy [5]–[7], where the AF strategy causes less delay and requires lower hardware complexity compared to other relay strategies. The authors in [5] deal with AF TWRNs with a single relay system. They derived closed form expressions of terminal and relay powers in order to minimize the total power consumption by respecting a certain received signal to noise ratio at the terminals. The work presented by Zhou et. al in [6] investigated the problem of the optimal number of relays to be selected in AF TWRNs with multiple relay systems. They showed that employing multiple relays is more energy efficient than using a single relay network. The performance of the network can be further improved by merging the principle of the AF TWRNs with Multi-Input Multi-Output (MIMO) antenna techniques. Indeed, MIMO techniques provide extra spatial dimensions to enhance the system performance. An overview of MIMO TWRNs using AF strategy has been presented in [8], [9]. The works in [8], [9] discussed the problem of MIMO TWRNs with a single relay. More specifically, in [9], the authors proposed two efficient antenna selection algorithms to solve a green optimization problem.

Most of the aforementioned papers on energy efficiency in TWRNs focus on the single or multi antenna system using a unique single relay only. However, to the best knowledge of the authors, the multiple relay problem with MIMO TWRNs has not been discussed so far. In this work, we investigate an energy efficient optimization problem for MIMO TWRNs using AF strategy with a multiple relay scheme. We therefore formulate in this paper an optimization problem for multiple relay MIMO TWRNs with AF strategy that minimizes the total power consumption of the network by taking into account the power budget at the relays and respecting a certain Quality-of-Service (QoS) at each terminal. That is, the data rate of each terminal is restricted to be greater than a pre-defined data rate threshold. More specifically, due to the non-convexity of the problem, we firstly derive a closed-form expression of the power allocated at each antenna at the terminal side for a fixed relay amplification gain. We then design a heuristic approach based on Particle Swarm Optimization (PSO) algorithm to find the optimal power allocated at each relay antenna. Details of the proposed method are given in Section III and Section IV.

The remainder of this paper is structured as follows. Section II investigates the system model. The total power minimization problem and optimal power solution are given in Section III. PSO algorithm is described in Section IV. Simulation results are discussed in Section V. Finally, the paper is concluded in Section VI.

Notations: The superscript $(\cdot)^T$ and $(\cdot)^H$ denote the transpose operator and the hermitian operator, respectively. $\mathbb{R}$ and $\mathbb{C}$ refer to the field of real and complex numbers, respectively. $\mathbb{E}(\cdot)$ and $\text{Tr}(\cdot)$ denote the expectation operator and the trace operator, respectively. $(x)^+$ denotes a maximum between $x$ and zero.

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II. System Model

Consider a cooperative MIMO TWRNs with two terminals $T_1$ and $T_2$ exchanging data via $L$ half-duplex relays using AF strategy in the absence of the direct $T_1$-$T_2$ link as shown in Fig.1. We assume that $T_1$ and $T_2$ are equipped by $M_{T_1}$ and $M_{T_2}$ antennas, respectively. We also assume that relays are equipped with the same number of antennas, i.e., $M_{R_i} = M_R, \forall i = 1, \ldots, L$.

We assume that the channel reciprocity between the $m^{th}$ terminal and the $i^{th}$ relay $H_{mri} \in \mathbb{C}^{M_R \times M_m}$ is constant during multiple times of transmissions. $h_{ab}^{x,y}$ are the corresponding elements representing the fading coefficients between transmit antenna $y$ at node $a$ and receive antenna $x$ at node $b$. Also, it is assumed that each terminal has full Channel State Information (CSI).

In our system model, we denote $V_m$ and $U_m$, where $m = 1, 2$, two unitary precoder and decoder matrices, respectively. In the first phase, $T_1$ and $T_2$ employ the precoder matrices $V_1$ and $V_2$, respectively, such as: $x_1 = V_1 \hat{x}_1$ and $x_2 = V_2 \hat{x}_2$, where $x_m$ is the transmitted signal after being precoded by $T_m$. Subsequently, during the second time slot, $T_1$ and $T_2$ employ the decoder matrices $U_2$ and $U_1$, respectively, such as: $r_{T_1} = U_2^H y_{T_1}$ and $r_{T_2} = U_1^H y_{T_2}$, where $y_{T_1}$ and $r_{T_2}$ are the received signals at $T_1$ before and after decoding, respectively, while $y_{T_2}$ and $r_{T_1}$ are the received signals at $T_2$ before and after decoding, respectively. The choice of $V_m$ and $U_m$ will be defined later.

In the first phase, $T_1$ and $T_2$ transmit their signals $\hat{x}_1$ and $\hat{x}_2$ to the relays simultaneously, with a power denoted $P_1 = [P_1^1, \ldots, P_1^M]$ and $P_2 = [P_2^1, \ldots, P_2^M]$, respectively, where $\mathbb{E} (|x_m|^2) = Tr (x_m^H x_m^H) = \sum_{q=1}^{M_m} P_m^q$, $m = \{1, 2\}$. Therefore, the complex baseband received signal at the $i^{th}$ relay can be written as

$$y_{r_i} = H_{1r_i} x_1 + H_{2r_i} x_2 + n_{r_i},$$

where $n_{r_i}$ is the transmitted signal after precoding by terminal $T_m$ and $n_{r_i}$ is the additive Gaussian noise vector at the $i^{th}$ relay. Without loss of generality, all the noise variances are assumed to be equal to $N_0$.

![Fig. 1 MIMO TWRNs system model.](image)

During the second phase, each relay amplifies $y_{r_i}$ by multiplying it by a diagonal matrix $W_i \in \mathbb{R}^{M_R \times M_R}$, with elements $w_i^{k}$ representing the amplification factor at the $k^{th}$ antenna of the $i^{th}$ relay and broadcasts it to the terminals. The $w_i^{k}$ factor is given by

$$|w_i^{k}|^2 = \frac{P_k^i}{\sum_{i=1}^{M_{r_i}} |h_{1r_i}^{k}|^2 + \sum_{i=1}^{M_{r_i}} |h_{2r_i}^{k}|^2 + N_0},$$

where $P_k^i$ denotes the power at the $k^{th}$ antenna of the $i^{th}$ relay. Hence, the received signals at $T_1$ and $T_2$ are given respectively by

$$y_{T_1}^* = A_2^H x_2 + A_2^H x_1 + z_1,$$

$$y_{T_2}^* = A_1^H x_2 + A_1^H x_1 + z_2,$$

where $A_2 = [A_2^1, \ldots, A_2^M]$ and $A_1 = [A_1^1, \ldots, A_1^M]$.

III. Total Power Minimization Problem and Optimal Power Expression

In this section, we aim to find the optimal power allocation at both terminals and relays in order to minimize the total power consumption of the network while satisfying the required QoS of both terminals. The received signals can be simplified by defining the precoding and decoding matrices basing on the Singular Value Decomposition (SVD) of the matrices $A_m$ where $A_m = U_m \Sigma_m V_m^H, m = 1, 2$. Thus, the total power minimization problem of MIMO TWRNs with multiple relays after SVD can now be formulated as

$$\min_{P_1, P_2, W} \mathcal{P} = \sum_{v=1}^{M_{T_1}} P_1^v + \sum_{u=1}^{M_{T_2}} P_2^u + \sum_{i=1}^{L} \sum_{k=1}^{M_{r_i}} \left( \sum_{v=1}^{M_{T_1}} |P_1^v h_{1r_i}^{k}|^2 ight)$$

$$+ \sum_{u=1}^{M_{T_2}} |P_2^u h_{2r_i}^{k}|^2 + N_0 |w_i^{k}|^2,$$

where $\mathcal{P}$ is defined as the total transmit power.
subject to

\[ 0 \leq \sum_{k=1}^{M_R} \left( \sum_{e=1}^{M_T} P_1^e |h_{1e}^k|^2 + \sum_{u=1}^{M_T} P_2^u |h_{2e}^k|^2 \right) N_0 |\phi_k^e|^2 \leq \bar{P}_r, \quad \forall i = 1, \ldots, L, \]

(C1: Power budget constraint at the relays):

\[ \frac{1}{2} \sum_{v=1}^{M_T} \log_2 \left( 1 + \frac{\sigma_{1v}^2 P_1^v}{C_{z_1}(v,v)\lambda} \right) \geq \beta R_{th}, \quad (10) \]

(C2: Lower bound constraint for the rate at \( T_1 \)):

\[ \frac{1}{2} \sum_{u=1}^{M_T} \log_2 \left( 1 + \frac{\sigma_{2u}^2 P_2^u}{C_{z_2}(u,u)\lambda} \right) \geq (1 - \beta) R_{th}, \quad (11) \]

(C3: Lower bound constraint for the rate at \( T_2 \)):

(C4: Positive power constraint):

\[ P_1 \geq 0, \quad P_2 \geq 0, \quad W \geq 0, \quad (12) \]

The utility function \( \mathcal{P} \) is none other than the sum of the power consumed by the terminals \( T_1 \) and \( T_2 \) in addition to the power consumed by all the relays. \( R_{th} \) is the lower sum rate bound required to maintain the QoS at the terminals, \( P_r \) is the peak power at each relay, and \( \sigma_{mq} \) is the \( q^{th} \) diagonal element of the matrix \( \mathbf{\Sigma}_m \). \( \beta \in [0, 1] \) is the rate profile of the system to characterize the boundary rates at the terminals.

Our optimization problem given in (8)-(12) is a non-convex problem and its optimal solution remains unsolved. In order to simplify the formulated optimization problem, we solve it in two steps; firstly, we assume that all the amplification factors of the relays are fixed (convert it to convex problem), i.e., minimizing the total power without any control on relay parameters. We then employ a heuristic approach to find the optimal amplification factors. Therefore, we can solve our convex optimization problem for fixed amplification factors by exploiting its strong duality [10].

\[
\max_{\lambda \geq 0} \min_{P_1 \geq 0, P_2 \geq 0} \mathcal{L}(\lambda, P_1, P_2)
\]

To solve this problem, we propose to use the Lagrangian method [10]. The Lagrangian expression is derived in (14). \( \lambda \) is the vector that contains all the Lagrangian multipliers of the system, where \( \lambda_r \) and \( \lambda_m \) represent the Lagrangian multipliers related to the peak power at the \( i^{th} \) relay and the Lagrangian multipliers related to the lower rate bound constraint at \( T_m \).

By taking the derivative of the Lagrangian with respect to the \( p_{m}^q \) where \( q = 1, \ldots, M_T, m = 1, 2 \), we can find the optimal transmit power allocated to the \( q^{th} \) antenna at the terminals that minimizes the Lagrangian function and the total power consumption of the network. The power expression at \( T_m \) is given in (15). We can employ the subgradient method, ellipsoid method, or other heuristic approaches to find the optimal Lagrangian multipliers of this problem (see [11]).

The challenge now is to find the optimal amplification factors at the relays. The idea is to adjust the terminal and relay powers simultaneously and iteratively until reaching an optimal solution.

IV. PARTICLE SWARM OPTIMIZATION ALGORITHM

PSO was introduced by Kennedy and Eberhart in 1995 [12]. This idea is inspired by swarm intelligence, social behavior, and food searching of a bird flocking and fish schooling. This approach can be widely used in several optimization fields due to its many advantages in terms of simplicity (i.e., few parameters to adjust) and low complexity implementation [13].

Algorithm 1 Particle Swarm Optimization Algorithm for MIMO TWRNs

1: Generate an initial population \( S \) composed of \( B \) random particles \( W^{(b)}, b = 1 \ldots B \).
2: while Not converged do
3: \hspace{1em} for \( b = 1, \ldots, B \) do
4: \hspace{2em} Find the optimal terminal power values (15) corresponding to the particle \( W^{(b)} \in \mathcal{S} \).
5: \hspace{2em} Compute the total power \( P_b \) using (8).
6: \hspace{1em} end for
7: \hspace{1em} Find \( (b_m, t_m) = \arg \min_b P_b(t) \) (i.e., \( b_m \) and \( t_m \) indicate the index and the position of the particle that results in the lowest total power).
8: \hspace{1em} Set \( P_{b_{\text{global}}} = P_{b_m}(t_m) \) and \( W^{b_{\text{global}}} = W_{b_m}(t_m) \).
9: \hspace{1em} Find \( t_b = \arg \min_b P_b(t) \) for each particle \( b \) (i.e., \( t_b \) indicate the position of the particle \( b \) that results in the lowest local total power).
10: \hspace{1em} Set \( P_{b_{\text{local}}} = P_{b(t_b)} \) and \( W^{b_{\text{local}}} = W_{b(t_b)} \).
11: \hspace{1em} Adjust the velocities and positions of all particles using equation (16) and (17).
12: \hspace{1em} \( t = t + 1 \).
13: end while

In PSO, we call each single solution “particle”. Firstly, the PSO generates randomly \( B \) particles \( W^{(b)}, b = 1, \ldots, B \), of length \( M_r \times L \) to form an initial population set \( S \). Secondly, the algorithm computes the total power consumed by the network (8) by each particle \( b \) using the optimal terminal power derived in (15). It then finds the particle that provides the global optimal total power consumption for this iteration, denoted \( W^{(b_{\text{global}})} \). In addition, it memorizes, for each particle \( b \), the position of its previous best performance, denoted \( W^{(b_{\text{local}})} \). After finding the two best values, PSO updates its velocity \( V^{(b)} \) and its position \( W^{(b)} \), respectively, at each iteration \( t \) as follows:

\[
V^{(b)}_j(t+1) = \phi_1 \left( W^{(b_{\text{local}})}_j(t) - W^{(b)}_j(t) \right) + \phi_2 \left( W^{(b_{\text{global}})}_j(t) - W^{(b)}_j(t) \right),
\]

\[
W^{(b)}_j(t+1) = \left( W^{(b)}_j(t) + V^{(b)}_j(t+1) \right)^+, \quad (17)
\]

where \( \phi_1 \) and \( \phi_2 \) are two random positive numbers generated for each \( j^{th} \) element of \( W^{(b)} \). This procedure is repeated until convergence (i.e., the optimal total power remains constant for a several number of iterations or the maximum number of iterations is reached). Details of the PSO algorithm applied to our formulated optimization problem of interest are given in Algorithm 1.
\[
L(\lambda, P_1, P_2) = \sum_{v=1}^{M_T} P_v^1 + \sum_{u=1}^{M_T} P_u^2 + \frac{1}{2} \sum_{v=1}^{M_T} \sum_{k=1}^{L_T} \left( \sum_{v=1}^{M_T} P_v^1 |h_{v,k}^v|^2 + \sum_{u=1}^{M_T} P_u^2 |h_{u,k}^u|^2 + N_0 \right) |w_k^v|^2 + \lambda_1 \left( \beta R_{th} - \frac{1}{2} \sum_{v=1}^{M_T} \log_2 \left( 1 + \frac{\sigma^2}{\bar{x}_{v,v}^m} \right) \right)
+ \lambda_2 \left( (1 - \beta) R_{th} - \frac{1}{2} \sum_{u=1}^{M_T} \log_2 \left( 1 + \frac{\sigma^2}{\bar{x}_{u,u}^m} \right) \right) + \frac{1}{2} \sum_{v=1}^{M_T} \lambda_v \left( \sum_{k=1}^{L_T} \sum_{v=1}^{M_T} P_v^1 |h_{v,k}^v|^2 + \sum_{u=1}^{M_T} |h_{u,k}^u|^2 + N_0 \right) |w_k^v|^2 - \bar{P}_r.
\]

\[
P_{m}^{q} = \left( \frac{\lambda_m}{(2 \log_2 2) \left[ 1 + \frac{1}{2} \sum_{v=1}^{M_T} \sum_{k=1}^{L_T} |h_{v,k}^v|^2 + \frac{1}{2} \sum_{k=1}^{L_T} \lambda_{2} \sum_{v=1}^{M_T} |h_{v,k}^v|^2 \right] - \frac{C_{x_{m}(q,q)}}{\sigma_{m,q}} \right) + m = \{1, 2\}. \tag{15}
\]

V. SIMULATION RESULTS

In this section, selected simulation and numerical results are presented for identically distributed Rayleigh fading channels. It is assumed that \(T_1\) and \(T_2\) are equipped with the same number of antennas (i.e., \(M_T = M_T = M\)). The noise power \(N_0\) is assumed to be equal to \(10^{-4}\). In all the following simulation results, “uniform” (extended scheme of equal power allocation presented in [5]) refers to the case when all terminal antennas have the same allocated power and all relay antennas have the same amplification factor (i.e., \(P_1 = P_1[1,\ldots,1], P_2 = P_2[1,\ldots,1]\) and \((W = w I)\) while “non-uniform” refers to the case when the optimal amplification factor is different from a relay antenna to another similarly to the power allocated to each terminal antenna.

![Fig. 2](image2.png)

Fig. 2 Total energy consumption \(P\) as a function of \(R_{th}\) for \(M_T = M_R = M, \beta = 0.5, L = 3, \bar{P}_r = 20\) dBm.

Fig.2 demonstrates the total power consumed by the network for both analytical and numerical solutions versus lower sum rate bound \(R_{th}\) for different values of \(M = \{2, 4\}\), random entries \([0,1]\) of \(W\), and fixed values of \(L\) and \(\bar{P}_r\). Note also that \(\beta = 0.5\). We can notice that the total power consumed is improving with the increase of \(M\) and this improvement is clearly observed for large values of \(R_{th}\). This shows the impact of introducing the MIMO scheme to get more degrees of freedom by increasing the number of antennas as such to reduce the power consumed. For instance, for \(R_{th} = 10\) Bits/s/Hz, we were able to reduce the total power consumed by around 38% by going from around 0.29 W to around 0.21 W using four antennas instead of two antennas. This

![Fig. 3](image3.png)

Fig. 3 Total energy consumption \(P\) as a function of \(R_{th}\) with \(M_T = M_R = M, \beta = 0.5, L = 3, \bar{P}_r = 20\) dBm for (a) \(L = 3\) and different values of \(M\), (b) \(M = 2\) and different values of \(L\).

In Fig.3, we aim to investigate the impact of the number of antennas and number of relays on the system performance. In this figure, we plot the total power consumed for both optimal solution and sub-optimal solution using PSO algorithm presented in Algorithm 1 versus \(R_{th}\) with different values of \(M\) and \(L\). It is shown that our sub-optimal solution achieves almost the same performance of the optimal one with a very small degradation when increasing \(L\) and/or \(M\) because of the increase of the possible candidate solutions. The initial system parameters are given as the following: \(\bar{P}_r = 20\) dBm, \(M = 2\), and \(L = 3\). In Fig.3(a), we plot the total power consumed by varying \(M\) and keeping \(L = 3\) fixed, while Fig.3(b) plots the total power consumed by varying \(L\) and keeping \(M = 2\) fixed.
As mentioned before, increasing the number of relays and/or the number of antennas will ensure more energy saving for the TWRNs by offering more diversity to relays and terminals to allocate their powers more efficiently.

![Fig. 4. Uniform and non-uniform power schemes versus $R_{th}$ for $\beta = 0.5$, $M = 2$, $L = 3$, and $P_t = 30$ dBm.](image)

Fig.4 depicts the sum power at the terminals, the average sum power at the relays, and total consumed power, and compare the later one with the uniform case for $M = 2$, $L = 3$, and $P_t = 30$ dBm. From this figure, we can deduce that the non-uniform solution provides significant advantages than uniform solution for different number of antennas and/or relays. For instance, when $R_{th} = 10$ Bits/s/Hz we can reduce the total power consumed by around 60% by having non-uniform case instead of optimal uniform case.

![Fig. 5. Non-uniforms power scheme versus $\beta$ for $R_{th} = 10$ Bits/s/Hz, $M = 2$, $L = 3$, and $P_t = 20$ dBm.](image)

Finally, Fig.5 illustrates the sum power variation at the terminals and at the relays with the total power versus $\beta$. One can see that the optimal $\beta$ to minimize the total power consumed by the network is equal 0.5 (corresponding to the symmetric point of the sum rate where the threshold rate at $T_1$ equal the threshold rate at $T_2$). At the beginning when $\beta < 0.5$, the power consumed by $T_2$ is greater than the power consumed by $T_1$. This can be justified by the fact that when $\beta < 0.5$, the lower rate bound at $T_1$ is less than the lower rate bound at $T_2$, thus, the terminal power required to satisfy constraint (11) is less that the terminal power required to satisfy constraint (11). The same interpretation is applied when $\beta > 0.5$, i.e., the power consumed by $T_1$ is greater than the power consumed by $T_2$. Concerning the power of the relays, we notice that it has a similar behavior due to the symmetry of the TWRNs (i.e., it is clear from (2) that the total power consumed in the relays follows the behavior of the total sum power consumed in the terminals).

VI. CONCLUSION

In this paper, the energy efficiency problem of multiple-input multiple-output two-way amplify-and-forward relay networks has been studied. Closed form analytical expressions of a transmit power allocation with fixed amplification factors have been derived. Moreover, particle swarm optimization algorithm was used to optimize the relay amplification factors simultaneously with the transmit power allocation in order to reach a near sub-optimal solution. It has been shown that the proposed method reaches a very close solution to the optimal one and that a non-uniform rate profile leads to a higher power consumption. In our ongoing task, we are working on applying our algorithm to series multi-hop two-way relay networks and studying the path-loss effect on the system.

REFERENCES