Near-Optimal Power Allocation with PSO Algorithm for MIMO Cognitive Networks using Multiple AF Two-Way Relays

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Abstract—In this paper, the problem of power allocation for a multiple-input multiple-output two-way system is investigated in underlay Cognitive Radio (CR) set-up. In the CR underlay mode, secondary users are allowed to exploit the spectrum allocated to primary users in an opportunistic manner by respecting a tolerated temperature limit. The secondary networks employ an amplify-and-forward two-way relaying technique in order to maximize the sum rate under power budget and interference constraints. In this context, we formulate an optimization problem that is solved in two steps. First, we derive a closed-form expression of the optimal power allocated to terminals. Then, we employ a strong optimization tool based on particle swarm optimization algorithm to find the power allocated to secondary relays. Simulation results demonstrate the efficiency of the proposed solution and analyze the impact of some system parameters on the achieved performance.

Index Terms—Multiple-input multiple-output, two-way cognitive radio, particle swarm optimization.

I. INTRODUCTION

Two-Way Relaying (TWR) is considered as one of the promising techniques in modern communication networks to increase the system throughput, enhance the network coverage area, and reduce the transmission power [1]. Traditional TWR consists of two transceivers exchanging different messages via a single/multiple relay(s) during two phases. At the receivers, TWR performs a self-interference cancelation to extract the desired message [2].

Recently, there has been a great attention from the wireless research community to combine the principle of the TWR using Amplify-and-Forward (AF) strategy with underlay Cognitive Radio (CR) in which secondary users utilize the frequency band allocated to primary users under some interference constraints to maintain a certain primary Quality-of-Service (QoS) [3].

Most of the previous works related to TWR-CR networks consider the single antenna scenario [4], [5]. The authors in [6] proposed an heuristic approach to reach a suboptimal solution of the power allocation and relay selection problems at the relay side for TWR-CR networks where all the communication nodes are equipped with a single antenna. However, Multiple-Input Multiple-Output (MIMO) technique provides more degrees of freedom to the system to enhance the system throughput. Various studies have employed MIMO system with TWR only [7], [8]. However, to the best knowledge of the authors, the multiple relay problem in TWR-CR networks with multiple antennas has not been discussed so far as it is the case in the non-cognitive case.

We consider a multiple relay scheme for MIMO TWR-CR networks employing AF strategy. In [9], we derived the expression of the optimal transmit powers allocated at each antenna of the communicating terminals assuming fixed amplification factors at the relays. In this work, we propose to enhance the achieved performance by optimizing the relay powers simultaneously with the terminal powers. Therefore, we formulate an optimization problem that maximizes the sum rate of the secondary network by considering the power budget at the communication terminals and relays in addition to the interference level tolerated by the primary user. An heuristic approach based on Particle Swarm Optimization (PSO) algorithm is proposed to solve the formulated optimization problem and achieve a near-optimal solution.

The remainder of this paper is organized as follows. Section II investigates the system model. The problem formulation is described in Section III. Closed-form expressions and the PSO algorithm are given in Section IV. Simulation results are discussed in Section V. Finally, the paper is concluded in Section VI.

Notations: The superscript $(\cdot)^T$ and $(\cdot)^H$ denote the transpose operator and the hermitian operator, respectively. $C$ and $\mathbb{R}$ refer to the field of complex and real numbers, respectively. $\mathbb{E}(\cdot)$ and $\text{Tr}(\cdot)$ denote the expectation and the trace operator. $(x)^+$ denotes a maximum between $x$ and zero.

II. SYSTEM MODEL

We consider one primary user and two cognitive terminal transceivers $T_1$ and $T_2$ exchanging their messages via $L$ cognitive half-duplex relays using TWR technique as shown in Fig.1. We assume that, the primary user, $T_1$, $T_2$, and the $i^{th}$ relay are equipped with $M_{PU}, M_{T1}, M_{T2}$ and $M_{RI}$ antennas, respectively. We employ the CR underlay mode in which the secondary users share the spectrum with the primary user...
by respecting a primary user tolerated interference threshold denoted $I_{th}$ [3]. Absence of the direct link between the secondary transceivers is also considered. Without loss of generality, all the noise variances are assumed to be equal to $N_0$.

During the first phase, $T_1$ and $T_2$ transmit their signals $\tilde{x}_1$ and $\tilde{x}_2$ to the relays at the same time, with a power denoted $P_1 = [P_1^1, ..., P_1^M]$ and $P_2 = [P_2^1, ..., P_2^M]$, respectively. In the second phase, the relays transmit the amplified signals to the terminals, with a power denoted $P_{ri} = [P_{r_1}, ..., P_{r_M}]$, where $i = 1, ..., L$. Let us define $P_1$ and $P_2$ as the peak powers at the transceiver terminals and at each relay, respectively. All the channel gains in our framework are assumed to be constant during the coherence time with elements $h_{ab}^{xy}$ representing the fading coefficients between transmit antenna $x$ and receive antenna $y$ at node $a$ and receive antenna $x$ at node $b$. In addition to that, channel reciprocity and perfect channel state information at transmitters and receivers are considered. Table I summarizes the system channel notations.

**TABLE I: SYSTEM MODEL CHANNEL NOTATIONS**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Complex channel mapping matrices between</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{mr_1}$</td>
<td>$C^{M_{R_1} \times M_{T_1}}$</td>
</tr>
<tr>
<td>$H_{mp}$</td>
<td>$C^{M_{T_1} \times M_{P_1}}$</td>
</tr>
<tr>
<td>$H_{r_1p}$</td>
<td>$C^{M_{P_1} \times M_{P_2}}$</td>
</tr>
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Let $V_m$ and $U_m$, where $m = 1, 2$, two unitary precoder and decoder matrices, respectively, employed by terminals. In the first phase, $T_m$ employs the precoder matrix such as: $x_m = V_m \tilde{x}_m$, where $x_m$ is the transmitted signal after being precoded by $T_m$. Subsequently, during the second phase, $T_1$ and $T_2$ employ the decoder matrices $U_2$ and $U_1$, respectively, such as: $r_1 = U_2^H y_1$ and $r_2 = U_1^H y_2$, where $y_1$ and $r_1$ are the received signals at $T_1$ before and after decoding, respectively, while $y_2$ and $r_2$ are the received signals at $T_2$ before and after decoding, respectively. The choice of $V_m$ and $U_m$ will be defined later. It is assumed that $\mathbb{E}(||x_m||^2) = \mathbb{E}(||x_m^H||^2) = \text{Tr}(x_m x_m^H) \leq P_t$.

### III. Problem Formulation

In this section, we formulate an optimization problem that maximizes the secondary sum rate for MIMO TWR-CR networks without affecting the QoS of the primary user. For simplicity, we assume that all the relays are equipped with the same number of antennas, i.e., $M_{R_i} = M_R, \forall i = 1, ..., L$.

In the first phase, the baseband received signal at the $i^{th}$ relay is given as follows

$$y_{ri} = H_{1ri} x_1 + H_{2ri} x_2 + n_{ri},$$

where $n_{ri}$ is the additive Gaussian noise vector at the $i^{th}$ relay and $x_m$ is the transmitted signal after precoding by terminal $T_m$. During the second phase, each relay amplifies $y_{ri}$, by multiplying it by a diagonal matrix $W_i \in \mathbb{R}^{M_R \times M_R}$ (containing the amplification factor $w_{ki}$ at each antenna $k$ of the $i^{th}$ relay) and broadcasts it to the terminals $T_1$ and $T_2$.

The amplification factor at the $k^{th}$ antenna of the $i^{th}$ relay can be expressed as

$$|w_{ki}|^2 = \frac{P_{ri}^k}{\sum_{x=1}^{P_{r_1}} ||h_{x,ki}^r||^2 + \sum_{z=1}^{P_{r_2}} ||h_{z,ki}^r||^2 + N_0},$$

where $P_{ri}^k$ denotes the power at the $k^{th}$ antenna of the $i^{th}$ relay. Finally, the received signals at terminals are given as

$$y_1 = \tilde{A}_2 x_1 + A_2 x_2 + z_1,$$

$$y_2 = A_1 x_1 + \tilde{A}_1 x_2 + z_2,$$

respectively, where $A_1 = \sum_{i=1}^{L} H_{2ri}^T W_i H_{1ri}, \tilde{A}_1 = \sum_{i=1}^{L} H_{2ri}^T W_i H_{2ri}, A_2 = \sum_{i=1}^{L} H_{1ri}^T W_i H_{2ri}, \text{ and } \tilde{A}_2 = \sum_{i=1}^{L} H_{1ri}^T W_i H_{1ri}$ are the equivalent MIMO channels obtained at the receivers. $z_m = \sum_{i=1}^{L} (H_{mir}^T W_i n_{ri}) + n_m$ and $n_m$ are the amplified noise and the additive Gaussian noise vectors at $T_m$, respectively, where $m = 1, 2$. By using the knowledge of the side information and channel reciprocity, the terminals can remove the self interference by eliminating their own signals (i.e. $x_1$ for $T_1$ and $x_2$ for $T_2$). The covariance matrix of the noise $z_m$ can be given as

$$C_{z_m} = \mathbb{E}[z_m z_m^H] = N_0 \sum_{i=1}^{L} H_{mir}^T W_i (H_{mir}^T W_i)^H + N_0 I,$$

where $I$ denotes the identity matrix and determined from the context. We then propose to define the precoding and decoding matrices using the Singular Value Decomposition (SVD) of the matrices $A_m$, where $m = 1, 2$. As such, the sum rate of the MIMO TWR after SVD can be written as

$$R = \frac{1}{2} \sum_{v=1}^{M_{R_1}} \log_2 \left(1 + \frac{\sigma_{1v}^2 P_{r_1}^i}{C_{z_1}(v, v)} \right)$$

$$+ \frac{1}{2} \sum_{u=1}^{M_{R_2}} \log_2 \left(1 + \frac{\sigma_{1u}^2 P_{r_2}^i}{C_{z_2}(u, u)} \right),$$

where $C_{z_1}(v, v) = \mathbb{E}[z_1(z_1^H z_1)]$ and $C_{z_2}(u, u) = \mathbb{E}[z_2(z_2^H z_2)]$ are the noise covariance matrices at terminals $T_1$ and $T_2$, respectively.
Finally, the sum rate maximization problem of MIMO TWR-CR with multiple relays can be formulated as

$$\max_{P_1, P_2, W} R(P_1, P_2, W)$$

subject to

$$0 \leq \sum_{v=1}^{M_{R_1}} P_1^v \leq \bar{P}_1, \quad 0 \leq \sum_{u=1}^{M_{R_2}} P_2^u \leq \bar{P}_2,$$

where $\sigma_{qq}$ is the $q^{th}$ diagonal element of the matrix $\Sigma_m$. The Lagrangian multipliers related to the peak power budget constraints (10) and (11) represent the peak power constraints at the cognitive transceivers, and at each cognitive relay, respectively. While the constraints (10) and (11) represent the interference constraints in the first and second phase interference constraints, respectively. By taking the derivative of the Lagrangian with respect to the $P^q_m$, where $q = 1 : M_{R_m}, m = 1, 2$, we find the optimal transmit power allocated to the $q^{th}$ antenna at the terminals that maximizes the Lagrangian function and, consequently, the sum rate. Its expression is given in (14).

We can employ the subgradient method, ellipsoid method, or other heuristic approaches to find the optimal Lagrangian multipliers of this problem [11]. Hence, to obtain the solution, we can start with any initial values for the different Lagrangian multipliers and evaluate the optimal powers. We then update the Lagrangian multipliers at the next iteration with a step size updated according to the nonsummable diminishing step length policy (see [11] for more details). The updated values of the optimal powers and the Lagrangian multipliers are repeated until convergence.

### B. Particle Swarm Optimization Algorithm

In the second step, we employ the PSO algorithm to optimize the terminal powers and amplification factors at each relay antenna, simultaneously. The PSO idea was introduced by Kennedy and Eberhart in 1995 [12]. This idea is inspired by swarm intelligence, social behavior, and food searching of a bird flocking and fish schooling. This approach can be widely used in several wireless communication fields due to its simplicity (i.e., few parameters to adjust) [13].

#### Algorithm 1 Particle Swarm Optimization Algorithm for MIMO TWR-CR networks

1. Generate an initial population $S$ composed of $B$ random particles $W^{(b)}$, $b = 1 \cdots B$.
2. while Not converged do
   3. for $b = 1, \cdots, B$ do
      4. Find the optimal terminal power by computing (14) corresponding to the particle $W^{(b)} \in S$.
      5. Compute the achieved sum rate $R_b$ using (6).
   6. end for
   7. Find $(b_y, t_y) = \arg \max_t R_b(t)$ (i.e., $b_y$ and $t_y$ indicate the index and the position of the particle that results in the highest sum rate).
   8. Set $R_{(b,\text{global})} = R_{b_y}(t_y)$ and $W^{(b,\text{global})} = W^{b_y}(t_y)$.
   9. Find $t_i = \arg \max_t R_b(t)$ for each particle $b$ (i.e., $t_i$ indicates the position of the particle $b$ that results in the highest local sum rate).
   10. Set $R_{(b,\text{local})} = R_b(t_i)$ and $W^{(b,\text{local})} = W^{b}(t_i)$.
   11. Adjust the velocities and positions of all particles using equations (15) and (16), respectively.
   12. Move to the next iteration $t = t + 1$.
3. end while

First, the PSO generates $B$ random particles (i.e., random amplification factor matrices $W^{(b)}$, $b = 1, \cdots, B$) of length $M_{R} \times L$ to form an initial population set $S$. The algorithm computes the achieved sum rate (6) of all particles by computing the optimal terminal powers (14) for this fixed amplification factor matrix $W^{(b)}$. It then finds the particle that provides the global optimal sum rate for this iteration, denoted $W^{(b,\text{global})}$. In addition, for each particle $b$, it memorizes the position of its previous best performance, denoted $W^{(b,\text{local})}$. After finding these two best values, PSO updates its velocity $V^{(b)}_j$ and its...
\[ L(\lambda, P_1, P_2) = \frac{1}{2} \sum_{i=1}^{M_T} \log_2 \left( 1 + \frac{\sigma^2_{x_{1i}} P_t}{\sigma^2_{x_{1i}} + N_0} \right) + \frac{1}{2} \sum_{u=1}^{M_T} \log_2 \left( 1 + \frac{\sigma^2_{x_{2u}} P_t}{\sigma^2_{x_{2u}} + N_0} \right) - \lambda_1 \left( \sum_{i=1}^{M_T} P_t^1 - P_t \right) - \lambda_2 \left( \sum_{u=1}^{M_T} P_t^2 - P_t \right) - \sum_{i=1}^{L} \lambda_i \left( \sum_{k=1}^{M_T} P_t^1 V_{1k}^2 + \sum_{u=1}^{M_T} P_t^2 V_{2u}^2 + N_0 \right) - \lambda_{th1} \left( \sum_{i=1}^{M_T} \sum_{j=1}^{M_T} P_t^1 |h_{ij}|^2 + \sum_{u=1}^{M_T} \sum_{j=1}^{M_T} P_t^2 |h_{2ji}|^2 - I_{th} \right) - \lambda_{th2} \left( \sum_{i=1}^{L} \sum_{j=1}^{M_T} \sum_{k=1}^{M_T} P_t^1 |h_{1ki}|^2 + \sum_{u=1}^{M_T} \sum_{k=1}^{M_T} P_t^2 |h_{2uk}|^2 + N_0 \right) \left( |h_{1ki}|^2 + |h_{2uk}|^2 - I_{th} \right) \]. (13)

\[ P_m^o = \left( \frac{1}{2 \log_2 \left( \lambda_m + \sum_{i=1}^{L} \lambda_i \sum_{k=1}^{M_p} |h_{ikr}|^2 + \lambda_{th1} \sum_{i=1}^{M_T} |h_{1ki}|^2 + \lambda_{th2} \sum_{i=1}^{L} \sum_{j=1}^{M_T} \sum_{k=1}^{M_T} |h_{1ki}|^2 |h_{2uk}|^2 \right) - \frac{\sigma^2_{w_i} \sigma_{w_j}}{\sigma_{w_i}} \right) \right) + , m = \{1, 2\}. (14) \]

Particle positions \( W_j^{(b)} \), respectively at each iteration \( t \) as follows:

\[ \begin{align*}
V_j^{(b)}(t+1) &= V_j^{(b)}(t) + \phi_1 \left( W_j^{(b, local)}(t) - W_j^{(b)}(t) \right) \\
&+ \phi_2 \left( W_j^{(b, global)}(t) - W_j^{(b)}(t) \right),
\end{align*} \] (15)

\[ W_j^{(b)}(t+1) = \left( W_j^{(b)}(t) + V_j^{(b)}(t+1) \right)^+, (16) \]

where \( \phi_1 \) and \( \phi_2 \) are two random positive numbers generated for each element \( j \). This procedure is repeated until convergence (i.e., sum rate remains constant for a several number of iterations or reaching maximum number of iterations). Details of the PSO algorithm as applied to our optimization problem of interest are given in Algorithm 1.

V. SIMULATION RESULTS

In this section, numerical results are provided for identically distributed Rayleigh fading channels. All the communication terminals of the system are equipped with the same number of antennas (i.e., \( M_{T_1} = M_{T_2} = M_{PU} = M_T \)). The noise variance \( N_0 \) is assumed to be equal to \( 10^{-2} \).

![Fig. 2. Achieved secondary sum rate versus peak terminal power constraint \( M_T = M_{TR} = M, L = 2 \) and \( P_t = 20 \) dBm for fixed \( W \) entries.](image)

![Fig. 3. Achieved secondary sum rate versus peak terminal power constraint with \( M_T = M_{TR} = M, L = 6 \), and \( P_t = 20 \) dBm, for (a) \( M = 1 \), (b) \( M = 4 \).](image)

Fig. 2 plots the achieved secondary sum rate versus terminals peak power \( (P_t) \) for different values of \( I_{th} = \{10, 20\} \) dBm with fixed entries of \( W \) chosen randomly between \([0, 1]\), and fixed value of \( L \), and \( P_t \). We can notice that the achievable rate is improving with the increase of \( P_t \) up to a certain point, due to the fact that starting from this value of \( P_t \) the system can no more supply the secondary terminals with their full power budgets because the system has to respect constraints (10) and (11). For this reason, we have introduced the MIMO scheme to get more degrees of freedom by increasing the number of antennas as such to enhance the secondary sum rate. For instance, for \( I_{th} = 20 \), we were able to double the achievable secondary sum rate by going from 6.5 bits/s/Hz to around 13 bits/s/Hz at high values of \( P_t \) by using four antennas instead of a single one. This figure shows also the validity of the derived closed-form solution expressed in (14) for fixed entries of \( W \) which is compared to the optimal solution obtained using simulations.
uniform" refers to the case where the amplification factor differs from one antenna to another. In this figure, we plot the achieved sum rate for different values of $I_{th}$ and $M$ with fixed $L = 6$ and $P_T = 20$ dBm versus $P_l$. One can see that, the proposed algorithm achieves almost the same performance of the optimal solution. Of course, increasing the system parameters (i.e., $M_T$, $M_R$, and/or $L$) may decrease the performance of the proposed method since in this case more possible solutions exist. For instance, the gap between both curves becomes greater with $M = 4$ as it is shown in Fig.3(b) mainly with the non-uniform case. It is trivial that optimizing all $W$ entries independently (i.e., non-uniform case) achieves better performance than assuming $W = wI$ (i.e., uniform case), however it requires a higher computational complexity as we are optimizing $M_R \times L$ variables instead of one variable.

Finally in Fig.4, we aim to investigate the impact of $M_T$, $M_R$, and $L$ on the system performance. The initial system parameters are given as the following: $P_l = 20$ dBm, $P_r = 10$ dBm, $I_{th} = 20$ dBm, $M_T = 2$, $M_R = 2$, and $L = 4$. In Fig.4(a), we plot the achievable sum rate for both optimal uniform (corresponding to the solid lines) and non-uniform (corresponding to the dashed lines) by varying one of the $\{M_T, M_R, L\}$ parameters and keeping all the others fixed. While Fig.4(b) plots the optimal uniform amplification factor $(w_{opt})$ that maximizes the sum rate. From this figure, we can deduce the following: (i) the non-uniform solution provides significant advantages than uniform solution for different number of antennas and/or relays, (ii) $M_T$ is the parameter that affects the most the system sum rate; as $M_T$ increases, the sum rate increases while with $M_R$ and $L$, we notice that the achieved sum rate is almost constant mainly for the uniform case (iii) For the uniform scenario, the higher $M_R$ and $L$, the lower the optimal amplification factor as the system has to satisfy both constraints (9) and (11), while the higher $M_T$, the higher $w_{opt}$ until a certain value (e.g., $M_T = 6$) since in this case the terminals can optimally adjust their powers over antennas without affecting all constraints.

VI. CONCLUSION

In this paper, we introduced and solved a new optimization problem for multiple-input multiple-output two-way relaying operating in underlay cognitive radio networks. More specifically, we consider multiple amplify-and-forward relays and optimized the relay amplification factors adaptively with the terminal transmit powers. Starting with a closed form analytical expression of a transmit power allocation with fixed amplification factors, we have used the particle swarm optimization algorithm as an heuristic approach to reach a near-optimal solution. Also, we have investigated the impact of some parameters on the system performance and showed that the number of terminal antennas is the most affecting parameter on the system sum rate. In our future work, we will study the system performance when imperfect channel state information is considered.

REFERENCES