Optimized Power Splitting Scheme for Multiple Energy Harvesting Two-Way Relays

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Abstract—This paper investigates a multiple relay selection scheme for Energy Harvesting (EH)-based two-way relaying. All the relays are considered as EH nodes that harvest energy from renewable and radio frequency sources, then use it to forward the information to the sources. The power-splitting (PS) protocol, in which the receiver splits the input radio frequency signal into two components: for information transmission and for energy harvesting, is adopted in the relay side. The objective is to optimize (i) the PS ratios, (ii) the relays’ transmit power, and (iii) the selected relays in order to maximize the total rate utility over multiple coherent time slots. A joint-optimization solution based on geometric programming and binary particle swarm optimization is proposed to solve the problem. Numerical results illustrate the behavior of the network versus various system parameters and compare the performance of the proposed approach with that of the dual-method and the branch-and-bound solutions.

Index Terms—Energy harvesting, multiple relay selection, power-splitting, two-way relaying.

I. INTRODUCTION

Two-Way Relaying (TWR) has lately attracted a great deal of interest due to its potential in achieving higher throughput with a lower power consumption [1]. Unlike the typical One-Way Relaying (OWR) transmission approach where four phases are needed to exchange different messages between two communicating terminals, the TWR requires only two phases. In the first phase, the sources transmit their signals simultaneously to relays which, in the second phase, broadcast the signal to the sources using one of the relaying strategies, e.g., Amplify-and-Forward (AF) and Decode-and-Forward (DF). The terminals, acting as receivers, apply a self-interference cancellation operation to extract the desired data [2].

On the other hand, nowadays, there has been a considerable interest in Energy Harvesting (EH) technique as one of the most robust method to perpetuate the lifetime and sustainability of wireless systems [3]. Many promising practical applications that can exploit this technique have been discussed recently, such as, emerging ultra-dense small cell deployments, point-to-point sensor networks, far-field microwave power transfer, and dense wireless networks [4].

One of the advantages of such techniques is to cope with the issues related to the supply of wireless devices located in remote or inaccessible areas such as sensors placed in forests or mountains where replenishing a new battery or recharging it using traditional wired techniques is not always possible. In addition, EH techniques, also known as energy scavenging techniques, enables networks’ owners to behave green towards the environment [5] as the devices will be powered by non-polluting alternative sources such as solar, wind, thermoelectric, or vibration [6]. Recently, Radio Frequency (RF)-based EH, also known as wireless energy transfer, has been introduced as an effective harvesting technology where energy is collected from RF signals generated by other neighbor devices. Unlike other Renewable Energy (RE) sources, the RF energy is widely available in the ambient atmosphere in all hours, days, and nights [3], [7].

Two protocols are proposed for the RF-based EH [8]. The Time Switching (TS) protocol where the receiver switches over time between the energy harvester equipment and the information decoder, and the Power Splitting (PS) protocol where a portion of the received signal is used for EH and the remaining for the information processing.

Most of the works existing in the literature investigated the RF-based EH for the traditional OWR technique [9], [10]. However, few studies dealt with RF-based EH with TWR but they are mainly focusing on the special case with one relay scheme. For instance, in [11], the authors studied the achievable RF TS throughput using AF relay without optimizing the total EH output for TWR system, while the authors of [12] focused on RE EH scheme considering that all nodes harvest energy only from RE sources where the power allocation of all the nodes for different relaying strategies are discussed.

In this framework, we investigate the TWR-AF scheme with PS protocol for multiple relays scenario. With the AF scheme, the relays amplify the received signal with a certain amplification gain before forwarding it to destinations. This allows a faster transmission without processing delay compared to other relaying strategies. The PS protocol is adopted in this paper as it outperforms the TS protocol mainly at high Signal-to-Noise Ratio (SNR) regime as shown in [13]. We consider that relays are simultaneously powered through RF signals and RE. The objective of the framework is to maximize the total throughput of the EH TWR system over a certain number of time slots while respecting the power budget and the storage capacity constraints at each relay. This is performed by determining, for each relay, its active or idle status, the fraction of signals to be harvested, and the amplification gain to be allocated for the broadcast phase. In this context, some of the relays can be turned to the idle mode such that they do not participate in the broadcasting process but continue harvesting energy from other transmitting (i.e., active) relays. Due to the non-convexity of the problem, we propose to
employ a joint-optimization approach to optimize the system parameters. A Binary Particle Swarm Optimization (BPSO) algorithm is adopted to find the set of active relays selected for data transmission. To optimize the other decision variables, we implement a Geometric Programming (GP) technique allowing to achieve a near-optimal solution to the problem [14]. The performance of the proposed approach is compared to that of the Branch-and-Bound-based (BB) solution in addition to the dual problem-based solution.

II. SYSTEM MODEL
A. System and Channel Models

We consider a half-duplex TWR system consists of two terminals, separated by a distance $D$ and denoted by $S_1$ and $S_2$, aiming to exchange information between each other through the help of multiple self-powered EH relays, denoted by $R_l, l = 1, \ldots, L$, placed within the communication range of both terminals.

The relays are placed within a circle centered in the middle of $S_1$ and $S_2$ with a radius equals to $\frac{D}{2}$ as shown in Fig. 1. Each node is equipped with a single antenna and $S_1$ and $S_2$ are not within the communication range of each other. In the TWR first phase, which is known as Multiple Access Channel (MAC) phase, both $S_1$ and $S_2$ send their messages $x_1$ and $x_2$ simultaneously to $R_l, \forall l = 1, \ldots, L$, with a power denoted by $P_l$ and $P_{2,l}$, respectively. In the second phase, which is known as Broadcast Channel (BC) phase, a set of relays are selected to amplify and forward the signal to the sources with a harvested power denoted by $P_{r_{1,b}}, \forall l = 1, \ldots, L, \forall b = 1, \ldots, B$.

We assume that the transmission will be performed in a finite period of time divided into $B$ blocks of equal size $T_c$, where $T_c$ is the time block or epoch length to exchange messages between $S_1$ and $S_2$.

We denote by $h_{1r_1,b}$ and $h_{2r_2,b}$ the channel gains during the $b$th block between $S_1$ and $R_l$ and between $S_2$ and $R_l$, respectively. The channel gain between $R_l$ and $S_1$ and between $R_l$ and $S_2$ (i.e., reverse link channels) are denoted by $h_{1r_1,b}^\ast$ and $h_{2r_2,b}^\ast$, respectively, where $(.)^\ast$ denotes the conjugate operator. The communication channel between two nodes $x$ and $y$ of the TWR system at time block $b$ is given as follows:

$$h_{xy,b} = \sqrt{d_{xy}^{-\alpha} h_{xy,b}},$$

where $d_{xy}$ is the Euclidean distance between the nodes $x$ and $y$, $\alpha$ is a pathloss constant, and $h_{xy,b}$ is a fading coefficient with a coherence time $T_c$ sec. Without loss of generality, all channel gains are assumed to be constant during the two transmission phases of TWR (i.e., one time block).

Although it is more important to investigate scenarios with causal channel state (i.e., the current and future channels are imperfectly known), in this study, we consider a simpler scenario assuming non-causal channel state known through prediction [15]. The results obtained in this paper constitute an upper bound for realistic scenarios and they provide a good insight on the behavior of the system over time. The analysis of imperfect channel state information scenarios are more elaborate and will be investigated in the future extension of this work. The transmitted signal power levels during each block $b$ are given as $E[|x_{1,b}|^2] = E[|x_{2,b}|^2] = 1$, where $E[\cdot]$ denotes the expectation operator.

B. Energy Harvesting Model

In this paper, two EH models are combined, i.e., the RE and RF models. We model the RE stochastic energy arrival rate as a random variable $\Phi$ Watt defined by a probability density function (pdf) $f(\varphi)$. For example, for photovoltaic energy, $\Phi$ can be interpreted as the received amount of energy per unit area. Note that, the half-duplex RF EH constraint, each node cannot harvest from RF and transmit simultaneously. On the other hand, each relay can harvest from RE during the whole period $T_c$. Note that, the non-selected relays remain silent and harvest energy during the whole period $T_c$ including the RF signal coming from the selected relays in the second information processing slot. The harvested energy is partially or totally stored to be used in future time blocks.

In this paper, $\varphi_{l,b}$ represents the instantaneous amount of RE produced during block $b$ at relay $l$, and $\mathcal{F}_b$ is the set of selected relays during block $b$. $\eta_{RF}$ and $\eta_{RE}$ denote the energy conversion efficiency coefficient of the RF and RE where $0 \leq \eta_{RF}, \eta_{RE} \leq 1$. A binary variable, denoted by $\epsilon_{l,b}$, is introduced to indicate the status of each relay where $\epsilon_{l,b} = 1$ if the relay is selected to amplify the signals, and $\epsilon_{l,b} = 0$, otherwise.

C. Relay Power Model

Since the energy arrivals and energy consumption are random and the energy storage capacities are finite, some relays might not have enough energy to serve users at a particular time. Under such scenario, it is preferred that some of the relays are kept OFF and allowed to recharge. Hence, each relay can be selected for transmission or not at each time block $b$. The decision of relays selection is made centrally, i.e., the decision is taken by a central entity based on the amounts of stored and consumed energy at each relay. The total power consumption of a relay, denoted by $P_{r_{1,b}}$, can be computed as follows:

$$P_{r_{1,b}} = a_0 + \begin{cases} a_{t} P_{r_{1,b}}, & \text{for transmission,} \\ a_r, & \text{for reception,} \end{cases}$$

where $a_0$, $a_t$, and $a_r$ correspond to the offset of site power which is consumed independently of the transmit power and is due to signal processing, battery backup, and cooling, the.
power consumption that scales with the radiated power due to amplifier and feeder losses, and the consumed power due to the power reception, respectively. \( P_{r,b} \) denotes the radiated power by relay \( r \) at a given time block \( b \).

III. Problem Formulation

In the MAC phase, the received signal at the \( l \)th relay during each \( T_c \) is given by

\[
y_{r_1,b} = \sqrt{P_1} h_{1r_1,b} x_{1,b} + \sqrt{P_2} h_{2r_1,b} x_{2,b} + n_{r_1,b},
\]

where \( n_{r_1,b} \) is the sum of two noises. An antenna additive Gaussian white noise (AWGN) at the \( l \)th relay during block \( b \) with variance \( \mathcal{N}_r \) and a noise introduced by the signal processing circuit from passband to baseband also assumed to be AWGN with zero mean and variance \( \mathcal{N}_0 \). In practice, the antenna noise has a negligible effect on the information signal and the average power of the received signal as well [16]. Hence, we ignore its impact in (3) (i.e., \( \mathcal{N}_r \ll \mathcal{N}_0 \)).

In the PS protocol, before transforming the received signal from passband to baseband, the relay uses part of it for EH and the remaining part for information transmission. Let us assume that \( \sqrt{1-\beta_{l,b}} \) is the relay \( l \) PS ratio during the \( b \)th block, where \( 0 \leq \beta_{l,b} \leq 1 \), such that \( \sqrt{1-\beta_{l,b}} \sqrt{P_1 h_{1r_1,b} x_{1,b} + \sqrt{P_2} h_{2r_1,b} x_{2,b}} \) corresponds to the part of RF signal that will be converted to a current, while the remaining part of the signal \( \beta_{l,b} \sqrt{P_1 h_{1r_1,b} x_{1,b} + \sqrt{P_2} h_{2r_1,b} x_{2,b}} \) is used for information processing as shown in Fig. 2. In this protocol, the transmission in each phase is performed during \( T_c/2 \).

The total harvested energy of the \( l \)th relay during block \( b \) for selected and non-selected relay, denoted by \( E_{l,b}^{h} \), is given as follows

\[
E_{l,b}^{h} = \begin{cases}
\epsilon_{l,b} \left( 1 - \beta_{l,b} \right) \left[ \gamma_{RF} \left( P_1 h_{1r_1,b}^2 + P_2 h_{2r_1,b}^2 \right) \right] \frac{T_c}{2} & \text{RF EH from sources} \\
+ \left( 1 - \epsilon_{l,b} \right) \left[ \gamma_{RF} \left( P_1 h_{1r_1,b}^2 + P_2 h_{2r_1,b}^2 \right) \right] \frac{T_c}{2} & \text{RF EH from sources} \\
+ \left[ \gamma_{RF} \sum_{j \in \mathcal{J}} P_{r,b} h_{r_1 r_j,b}^2 \right] \frac{T_c}{2} & \text{RF EH from selected relays} \\
\end{cases}
\]

where \( E_{l,e} \) is the leakage energy within block \( b \). \( E_{l,b}^{c} \) corresponds to the consumed energy by relay \( l \) during block \( b \) due to information processing and is given as:

\[
E_{l,b}^{c} = a_0 T_c + \epsilon_{l,b} \left( a_r + a_t P_{r,b} \right) \frac{T_c}{2} + \left( 1 - \epsilon_{l,b} \right) \left[ a_r T_c \right].
\]

Note that, initially, we assume that the battery of relay \( l \) may already have a certain amount of charge denoted by \( B_{r,l,0} \) (i.e., \( E_{l,0} = B_{r,l,0} \)). During the BC phase, the selected relays amplify the received signal by multiplying it by the relay amplification gain denoted by \( w_{l,b} \). Then, they broadcast it to \( S_1 \) and \( S_2 \). Hence, the received signals at \( S_1 \) and \( S_2 \) at block \( b \) are given, respectively, as

\[
y_{1,b} = \sum_{l=1}^{L} \epsilon_{l,b} h_{1r_1,b} x_{1,b} + \sqrt{P_1} h_{1r_1,b} \sqrt{F_1 x_{1,b}} + n_{1,b},
\]

\[
y_{2,b} = \sum_{l=1}^{L} \epsilon_{l,b} h_{2r_1,b} x_{2,b} + \sqrt{P_1 h_{1r_1,b}^2 + P_2 h_{2r_1,b}^2} + n_{2,b},
\]

where \( n_{1,b} \) and \( n_{2,b} \) are the AWGN noise with zero mean and variance \( \mathcal{N}_0 \) at the receivers \( S_1 \) and \( S_2 \), respectively. The amplification gain at the relay \( l \) during block \( b \) can be expressed as:

\[
w_{l,b} = \frac{P_{r,b}}{\beta_{l,b} (P_1 h_{1r_1,b}^2 + P_2 h_{2r_1,b}^2) + \mathcal{N}_0},
\]

In (9), we ignore the noise effect in the denominator [17]. Without loss of generality, this approximation simplifies the subsequent derivations without having a significant impact on the achieved results. Therefore, the TWR sum-rate during the block \( b \) can be expressed as

\[
R_b = \frac{T_c}{2} \sum_{q=1}^{2} \log_2 (1 + \gamma_{b,q}) + \mathcal{N}_0
\]

where \( \gamma_{b,q} \) is given as follows:

\[
\gamma_{b,q} = \frac{P_{q} \left( \sum_{l=1}^{L} \epsilon_{l,b} w_{l,b} \sqrt{\beta_{l,b} h_{q_{r_1,b} h_{q_{r_1,b}}}} \right)^2,}
\]

where \( \bar{q} = 1 \) if \( q = 2 \) and vice versa. Consequently, the optimization problem maximizing the TWR sum-rate, denoted by \( R \), while satisfying the energy consumed and stored constraints for EH with PS protocol using AF is given as:

\[
\text{maximize } \epsilon_{l,b}, P_{l,b} \geq 0 \quad R = \sum_{b=1}^{B} R_b
\]

subject to:

\[
E_{l,b}^{h} + E_{l,b}^{c} \leq E_{l,b}^{h-1}, \quad \forall b = 1, \ldots, L, \forall b = 1, \ldots, B,
\]

\[
E_{l,b-1}^{h} + E_{l,b}^{h} \leq N \quad \forall b = 1, \ldots, L, \forall b = 1, \ldots, B,
\]

\[
0 \leq P_{r,b} \leq \bar{P}_r, \quad \forall b = 1, \ldots, L, \forall b = 1, \ldots, B,
\]

\[
0 \leq \beta_{l,b} \leq 1, \quad \forall b = 1, \ldots, L, \forall b = 1, \ldots, B,
\]

\[
\epsilon_{l,b} \in \{0, 1\}, \quad \forall b = 1, \ldots, L, \forall b = 1, \ldots, B,
\]
where \( \epsilon = [\epsilon_{l,b}]_{L \times B}, \beta = [\beta_{l,b}]_{L \times B}, \) and \( P_{r} = [P_{r,l,b}]_{L \times B} \) are matrices containing the relay status, the PS ratios, and the relay transmit power levels of each block \( l \) at each block \( b \), respectively. Constraint (12) ensures that the consumed energy during block \( b \) for any relay is always less than or equal to the stored energy at block \( b-1 \). Constraint (13) indicates that the energy stored at a relay cannot exceed the capacity of its super-capacitor at any time. Constraints (14) and (15) indicate the transmit power and PS ratio limits.

IV. JOINT-OPTIMIZATION SOLUTION

Due to the non-convexity of the optimization problem formulated in (11)-(16), we propose to proceed with a joint-optimization approach where we optimize the binary matrix \( \epsilon \) using the PSO algorithm and the other continuous decision variables (\( \beta \) and \( P_{r} \)) using GP. For a fixed and known \( \epsilon \), we apply a successive convex approximation (SCA) approach to transform the non-convex problem into a sequence of relaxed convex subproblems to approximated solution [14], [18].

A. Geometric Programming Method

GP is a class of nonlinear and nonconvex optimization problems that can be efficiently solved after converting them to a nonlinear but convex problems [19]. The interior-point method can be applied to GP with a polynomial time complexity [19]. The standard form of GP is defined as the minimization of a posynomial function subject to inequality posynomial constraints and equality monomial constraints, where a monomial is defined as a function \( f : \mathbb{R}^{n}_{++} \rightarrow \mathbb{R} \) where for each input vector \( x \), we associate \( f(x) = dx_{1}^{\alpha_{1}}x_{2}^{\alpha_{2}}...x_{n}^{\alpha_{n}} \), where \( x_{i} \) are the elements of \( x \), \( d \) is a non-negative multiplicative constant \( d \geq 0 \), and \( \alpha_{i} \in \mathbb{R}, i = 1, ..., n \) are the exponential constants. A posynomial is a non-negative monomial of sums.

In general, GP in its standard form is a non-convex optimization problem, because posynomials and monomials functions are not convex functions. However, with a logarithmic change of the variables, objective function, and constraint functions, it can be turned into an equivalent convex form using the property that the logarithmic sum of exponential function is a convex (see [19] for more details). From a relaxed GP, we propose an approximation to solve out the original non-convex problem. Therefore, for a given \( \epsilon \), we transform the objective function as follows:

\[
\minimize_{z \geq 0} \prod_{b=1}^{B} \prod_{q=1}^{2} \frac{1}{1 + \gamma_{b,q}} \equiv \minimize_{z \geq 0} \prod_{b=1}^{B} \prod_{q=1}^{2} \frac{1}{1 + \gamma_{b,q}},
\]

where \( z \triangleq [\beta, P_{r}] \). For notational convenience, let us define:

\[
\delta_{l,b,q}^{(1)} = \frac{\epsilon_{l,b} |h_{qr1,b}^{2} + P_{2} |h_{qr2,b}^{2}|^{2}}{P_{1} |h_{qr1,b}^{2} + P_{2} |h_{qr2,b}^{2}|^{2}}, \delta_{l,b,q}^{(2)} = \frac{\epsilon_{l,b} |h_{qr1,b} h_{qr2,b}|^{2}}{\sqrt{P_{1} |h_{qr1,b}|^{2} + P_{2} |h_{qr2,b}|^{2}}.
\]

Hence, after some manipulations, (17) can be re-expressed as:

\[
\minimize_{z \geq 0} \prod_{b=1}^{B} \prod_{q=1}^{2} \frac{1}{1 + \gamma_{b,q}} \leq \minimize_{z \geq 0} \prod_{b=1}^{B} \prod_{q=1}^{2} \frac{1}{1 + \frac{z}{\delta_{l,b,q}^{(1)} \gamma_{b,q} B_{l,b}}}
\]

It can be noticed from (19) that \( f_{l,b,q}(z) \) and \( g_{l,b,q}(z) \) are posynomials, however the ratio is not necessary a posynomial. Therefore, in order to convert the objective function to a posynomial, we propose to apply the single condensation method to approximate the denominator posynomial \( g_{l,b,q}(z) \) to a monomial function, denoted by \( \tilde{g}_{l,b,q}(z) \), using the arithmetic-geometric mean inequality as a lower bound [14]. Given the value of the \( z \) at the iteration \( i-1 \) of the SCA \( z^{(i-1)} \), the posynomial \( g_{l,b,q}(z) \), that, by definition, has the form \( \tilde{g}_{l,b,q}(z) \triangleq \sum_{k=1}^{K} \mu_k(z) \), where \( \mu_k(z) \) are monomials, can be approximated as:

\[
g_{l,b,q}(z) \leq \tilde{g}_{l,b,q}(z) = \prod_{k=1}^{K} \left( \frac{\mu_k(z)}{\tilde{g}_{k}(z^{(i-1)})} \right)^{\phi_k(z^{(i-1)})},
\]

where \( \phi_k(z^{(i-1)}) = \frac{\mu_k(z)^{2(i-1)}}{\tilde{g}_{k}(z^{(i-1)})} \). \( K = (L + 1)(L + 2)/2 \) corresponds to the total number of monomials in \( g_{l,b,q}(z) \) given in (19). It can be seen that the objective function is now posynomial because posynomial over monomial is posynomial and the product of posynomials is posynomial. Next, we apply the same approximations to the inequality constraints to obtain posynomials and fit into the GP standard form. Let us define the following:

\[
\zeta_{l,b}^{(1)} = \epsilon_{l,b} \frac{\eta_{RF}(P_{1} |h_{r1,b}^{2} + P_{2} |h_{r2,b}^{2}|^{2})}{T_{c}},
\]

\[
\zeta_{l,b}^{(2)} = \frac{1 - \epsilon_{l,b}}{\eta_{RF} T_{c}},
\]

\[
\zeta_{l,b}^{(3)} = \epsilon_{l,b} \frac{\eta_{RF}(P_{1} |h_{r1,b}^{2} + P_{2} |h_{r2,b}^{2}|^{2})}{T_{c}} + (1 - \epsilon_{l,b}) \frac{\eta_{RF}(P_{1} |h_{r1,b}^{2} + P_{2} |h_{r2,b}^{2}|^{2})}{T_{c}} + \frac{\eta_{RF} \varphi_{l,b}}{T_{c}},
\]

\[
\theta_{l,b}^{(2)} = \frac{\epsilon_{l,b} \alpha_{t} T_{c}}{2},
\]

\[
\theta_{l,b}^{(2)} = \alpha_{t} + \epsilon_{l,b} \frac{T_{c}}{2} + (1 - \epsilon_{l,b}) \frac{\alpha_{t} T_{c}}{2}.
\]

Thus, \( E_{1,b}^{h} \) and \( E_{1,b}^{c} \) given in (4) and (6), can be, respectively, expressed as:

\[
E_{1,b}^{h} = -\zeta_{l,b}^{(1)} \beta_{l,b} + \zeta_{l,b}^{(2)} \sum_{j \in J_b} P_{r,j} |h_{r1,j,b}^{2} + \zeta_{l,b}^{(3)} \right),
\]

\[
E_{1,b}^{c} = \theta_{l,b}^{(1)} P_{r,b} + \theta_{l,b}^{(2)}.
\]

By expanding \( E_{1,b-1}^{c} \), constraint (12) can be re-written as:

\[
\sum_{t=1}^{K} \left( \frac{\phi_{l,t} P_{r,j} |h_{r1,j,t}^{2} + \zeta_{l,t}^{(1)} \beta_{l,t} \right) \leq 1, \quad \forall l, \forall b.
\]
The equivalent constraint given in (24) is a posynomial over posynomial. Therefore, we can use the same approximation used in (20) to lower bound the denominator in (24) by \( \tilde{v}_{t,b}(z) \) with a total number of monomials \( K = \sum_{l=1}^{b}(|J_l| + 1) \). Similarly, we can rewrite constraint (13) as follows:

\[
\sum_{l=1}^{b} \left( \sum_{j \in J_l} \left( \psi_{l,t}^{(2)} \sum_{j \in J_l} P_{r_t,r_{j,t}} |h_{r_t,r_{j,t}}|^2 + \psi_{l,t}^{(3)} \right) \right) \leq 1, \quad \forall l, \forall b.
\]

The same approximation used in (20) to lower bound the denominator in (24) by \( \tilde{v}_{t,b}(z) \) and \( K = 4b - 2 \).

By considering the approximation of (20), (24), and (25) and given a fixed value of \( \epsilon \), we can formulate a GP approximated subproblem at the \( i^{th} \) iteration of the SCA for the considered optimization problem given in (11)-(16) as follows:

\[
\min_{\mathbf{z}, \epsilon \geq 0} \prod_{b=1}^{B} \prod_{q=1}^{2} f_{l,b,q}(z) \prod_{l,b} \frac{g_{l,b,q}(z)}{\tilde{v}_{t,b}(z)} \quad \text{subject to:}
\]

\[
\sum_{l=1}^{b} \left( \sum_{j \in J_l} \left( \psi_{l,t}^{(1)} P_{r_t,r_{j,t}} + \psi_{l,t}^{(2)} + E_{l,e} + \psi_{l,t}^{(3)} \beta_{l,t} \right) \right) \leq 1, \quad \forall l, \forall b,
\]

\[
\sum_{l=1}^{b} \left( \sum_{j \in J_l} \left( \psi_{l,t}^{(2)} P_{r_t,r_{j,t}} |h_{r_t,r_{j,t}}|^2 + \psi_{l,t}^{(3)} \beta_{l,t} \right) \right) \leq 1, \quad \forall l, \forall b,
\]

\[
P_{r_t,b} \leq 1, \quad \forall l = 1, \ldots, L, \forall b = 1, \ldots, B,
\]

(15), (16)

Hence, this optimization problem can be solved at each iteration of the SCA as given in Algorithm 1.

### Algorithm 1 SCA Algorithm

1: \( i = 1 \).
2: Select a feasible initial value of \( \mathbf{z}^{(i)} = [\beta^{(i)}, \mathbf{P}^{(i)}] \).
3: \text{repeat}
4: \( i \leftarrow i + 1 \).
5: Approximate the denominators of (19), (24), and (25) using the arithmetic-geometric mean as indicated in (20) using \( \mathbf{z}^{(i-1)} \).
6: Solve the optimization problem (26)-(29) using the interior-point method to determine the new approximated solution \( \mathbf{z}^{(i)} = [\beta^{(i)}, \mathbf{P}^{(i)}] \).
7: \text{until No improvement in the objective function.}

### B. Binary Particle Swarm Optimization

In order to optimize the binary matrix \( \epsilon \), we propose to employ the BPSO algorithm to reach a near-optimal solution of the problem. The BPSO algorithm was first developed in 1997 by J. Kennedy and R. Eberhart [20]. The idea is inspired from swarm intelligence, social behavior, and food searching by a flock of birds and a school of fish. BPSO presents several advantages compared to the other meta-heuristic approaches. Hence, we choose to apply it in the joint-optimization approach. The main advantages are summarized as follows:

(i) BPSO presents a simple search process and is easy to implement with few parameters to manipulate (e.g., such as the number of particles and acceleration factors for BPSO), (ii) it requires low computational cost attained from small number of agents, and (iii) it provides a good convergence speed [21].

The BPSO starts by generating \( T \) particles \( \epsilon^{(t)}, t = 1 \cdots T \) of size \( L \times B \) to form an initial population \( S \). Then, it determines the utility \( U \) (i.e., sum-rate) achieved by each particle by solving the optimization problem using GP approach developed in IV-A (or the dual-method for comparison purpose in the simulation results section). Then, it finds the particle that provides the highest solution for this iteration, denoted by \( \epsilon^{\text{max}} \). In addition, for each particle \( t \), it saves a record of the position of its previous best performance, denoted by \( \epsilon^{(t,\text{local})} \). Then, at each iteration \( i \), BPSO computes a velocity term \( V_{l,b}^{(t)} \) as follows:

\[
V_{l,b}^{(t)}(i) = \Omega V_{l,b}^{(t)}(i-1) + \psi_1(i) \left( \epsilon_{l,b}^{(t,\text{local})}(i) - \epsilon_{l,b}^{(t)}(i) \right) + \psi_2(i) \left( \epsilon_{l,b}^{\text{max}}(i) - \epsilon_{l,b}^{(t)}(i) \right),
\]

(30)

where \( \Omega \) is the inertia weight and \( \psi_1 \) and \( \psi_2 \) are two random positive numbers (\( \psi_1, \psi_2 \in [0, 2] \)) generated for each iteration \( i \) [20]. Then, it updates each element \( i \) of a particle \( \epsilon^{(t)} \) as follows:

\[
\epsilon_{l,b}^{(t)}(i+1) = \begin{cases} 1 & \text{if } r_{\text{rand}} < \Phi \left( V_{l,b}^{(t)}(i) \right), \\ 0 & \text{otherwise.} \end{cases}
\]

(31)

where \( r_{\text{rand}} \) is a pseudo-random number selected from a uniform distribution in \([0, 1]\) and \( \Phi \) is a sigmoid function for transforming the velocity to probabilities and is given as:

\[
\Phi(x) = \frac{1}{1 + e^{-x}}.
\]

(32)

These steps are repeated until reaching convergence by either attaining the maximum number of iterations or stopping the algorithm when no improvement is noticed. Details of the joint-optimization approach are given in Algorithm 2.
In this section, selected numerical results are provided to evaluate the performance of the PS protocol with multiple EH relays in TWR systems. $S_1$ and $S_2$ transmit their messages periodically every $T_c = 1$ for $L = 3$ and $B = 8$ time blocks unless otherwise stated. All the fading channel gains adopted in the framework are assumed to be independent and identically distributed (i.i.d) Rayleigh fading gains. The relays are randomly placed inside a circle centered in the middle of $S_1$ and $S_2$ with a distance $D = 50$ meters unless otherwise stated. The noise variance and the efficiency conversion ratios are stated. The noise variance and the efficiency conversion ratios $\epsilon$ and scale parameter $\theta$ are set to $4$ and $8$, respectively. For simplicity, we assume that $P_1 = P_2 = P_s$. The relay power parameters are given as: $a_0 = 1.2$ W, $a_r = 1.2$ mW, and $a_t = 4$ mW. At each relay, $RE$ is assumed to be generated following a Gamma distribution with shape parameter $k = 0.5$ and scale parameter $\theta = 1$. $RE$ is generated such that the constant power consumption of the relays, i.e., namely $a_0$, is handled. In other words, the transmit power consumption is covered by the harvested RF energy in addition to the available extra RE. The total stored energy cannot exceed $E_s = 5$ J and the battery leakage is set to be $E_{le} = 10$ mJ every b. A Monte Carlo simulation with 5000 iterations is performed to determine the average performance of the investigated TWR system using the BPSO-based solution given in Algorithm 2.

The BPSO is executed with the following parameters: $T = 12$ and $\Omega \in [0, 1]$ is a linear decreasing function of the BPSO iterations expressed as follows: $\Omega = 0.9 - \frac{t(0.9 - 0.2)}{I}$, where $I = 200$ is the maximum number of iterations. The joint-optimization approach solution using BPSO is compared to three other approaches: a BB-based solution with GP, a BPSO-based solution with the dual method, and a BB-based solution with the dual method. Note that, for a given $\epsilon$, the dual solution corresponds to the solution obtained by solving the dual problem of the primal problem given in (11)-(15). The corresponding solution represents a lower-bound of the optimal one due to the non-convexity of the problem (i.e., weak duality). On the other hand, the BB method achieves an optimal solution with respect to $\epsilon$ but it requires a very high computational complexity [22].

In Table I, we study the behavior of the TWR system for a given channel realization, a relay power budget $\bar{P}_r = 20$ dBm, and a source transmit power $P_s = 10$ dBm. In this table, we consider the total sum-rate as objective function and we compare to a utility maximizing the minimum rate of all time blocks denoted by “max-min”. With the max-min utility, the problem is solved using the same approach employed in this paper, however, the corresponding derivations are omitted due to space limitation. The objective is to study in details the advantages and disadvantages of each utility function and the differences in the corresponding decision variables. It can be noticed that the use of max-min utility helps in avoiding low rates achieved in certain blocks with the sum utility such as the rates in blocks 1 and 2: $R_1 = 1.55$ bps/Hz and $R_2 = 0.7$ bps/Hz. However, this advantage is compensated by a lower total sum-rate over the blocks. With sum utility, the system prefers to harvest more RF energy in order to exploit it during next time slots to achieve higher rates. For instance, it achieves $R_3 = 9.11$ bps/Hz and $R_4 = 10.07$ bps/Hz with sum utility instead of $R_3 = 6.92$ bps/Hz and $R_4 = 4.36$ bps/Hz with the max-min one.

In Fig. 3(a), we compare between the performance of the two utility metrics by plotting the average sum-rate versus the terminals’ power $P_s$ for a TWR system transmitting messages using the same parameter as Table I. The proposed joint-optimization approach is compared to dual solution employed jointly with BPSO. Obviously, as $P_s$ increases, the total sum-rate increases up to a certain value. In fact, increasing $P_s$ allow relays to harvest more RF energy and, at the same time, contribute to the rate improvement. The average results in Fig 3(a) confirms those of Table I, the sum-rate utility reaches higher performance than that of the max-min one. On the other hand, we notice an important gap achieved thanks to the use of the GP method instead of the dual method. Although GP is not optimal, it clearly achieves a near-optimal solution. In Fig. 3(b), we investigate the pathloss effect on the system performance by varying the distance $D$ separating the sources from 25 to 200 meters with the system parameters used in Fig 3(a). Fig 3(b) shows that the achieved throughput is decreasing while increasing distance $D$. This is due to the pathloss effect on both the SINR and the amount of harvested RF. Notice that, for large distances, the achieved sum-rate is mainly due to the extra RE generated. Indeed, as it is shown in Fig. 3(c), RF energy is no more available for power transmission as the harvested energy is almost zero. This confirms that RF energy harvesting is only applicable within ultra-dense wireless networks. Fig. 3(c) also shows that high values of terminals’ transmission power $P_t$ helps in producing more RF energy.

In Fig. 4, we investigate the impact of the relay power budget $\bar{P}_r$ on the achieved sum-rate. Similar to Fig. 3, as $\bar{P}_r$ increases, the sum-rate increases up to a certain level where the TWR system becomes limited by the power budget at the sources. We also compare between the performance of the proposed joint-optimization approach (GP with BPSO) with those of GP with BB, dual solution with BPSO, and dual-solution with BB. We can clearly deduce that BPSO is able to achieve close performance to that of the solution obtained with BB while presenting a much lower complexity compared to that of BB. Furthermore, GP enables the achievement of better solution than the dual problem-based optimization one.

Finally, in Fig. 4, we compare the proposed approach with another suboptimal scenario where all $P_{r,b}$ are chosen to be fixed and constant ($P_{r,b} = \bar{P}_r$). This is performed to show the importance of the optimization of the relay transmit power levels simultaneously with the PS ratios and its impact on the
VI. CONCLUSIONS

In this paper, we proposed a multiple relay selection scheme for PS protocol-based energy harvesting two-relaying system. The relays harvest energy from renewable energy and radio frequency sources. We formulated an optimization problem aiming to maximize the total sum-rate over multiple time blocks. Due to the non-convexity of the optimization problem, we adopted a joint-optimization approach based on binary particle swarm optimization and geometric programming. The proposed solution enables the system to achieve near optimal solutions with a significant gain compared to dual problem-based solution. The behavior of the TWR system is studied via multiple numerical simulations.

REFERENCES