A Low Complexity Algorithm for Multiple Relay Selection in Two-Way Relaying Cognitive Radio Networks

Ahmad Alsharoa, Hakim Ghazzai, and Mohamed-Slim Alouini,
Computer, Electrical, and Mathematical Science of Engineering (CEMSE) Division,
King Abdullah University of Science and Technology (KAUST),
Thuwal, Makkah Province, Kingdom of Saudi Arabia,
E-mails: {ahmad.sharoa, hakim.ghazzai, slim.alouini}@kaust.edu.sa.

Abstract—In this paper, a multiple relay selection scheme for two-way relaying cognitive radio network is investigated. We consider a cooperative Cognitive Radio (CR) system with spectrum sharing scenario using Amplify-and-Forward (AF) protocol, where licensed users and unlicensed users operate on the same frequency band. The main objective is to maximize the sum rate of the unlicensed users allowed to share the spectrum with the licensed users by respecting a tolerated interference threshold. A practical low complexity heuristic approach is proposed to solve our formulated optimization problem. Selected numerical results show that the proposed algorithm reaches a performance close to the performance of the optimal multiple relay selection scheme either with discrete or continuous power distributions while providing a considerable saving in terms of computational complexity. In addition, these results show that our proposed scheme significantly outperforms the single relay selection scheme.

Index Terms—Cooperative cognitive radio network, two-way relaying, amplify and forward, multiple relay selection.

I. INTRODUCTION

Cognitive Radio (CR) and cooperative communication provide smart solutions to improve the spectrum usage and the data rate. CR was introduced recently as a promising technique to increase the spectrum efficiency in wireless communication [1]. CR spectrum sharing allows Secondary Users (SUs) known also as unlicensed users to access the frequency band allocated by Primary Users (PUs) known also as licensed users. As such and in order to protect the PUs, the sum of the interference power due to the Secondary Network (SN) should be kept below a certain tolerance called the interference temperature limit. Several studies have been proposed to analyze CR with cooperative relaying technique to increase the throughput of wireless networks [2]–[6].

The aforementioned works focused on unidirectional transmission known also as One-Way Relaying (OWR). The work presented by Li et. al in [2] investigates the joint single relay selection problem to find the optimal power allocation. They also proposed a low complexity approach to maximize the system throughput. In [4]–[6], heuristic multiple relay selection algorithms for OWR-CR network are investigated. More specifically, the works in [4] and [5] investigate the single PU case while in [6] the authors deal with multiple PUs case.

On the other front, there has been recently a great deal of interest in bidirectional transmission known also as Two-Way Relaying (TWR). In conventional TWR, the transmission process takes place in two phases. In the first phase, the source and the destination transmit their signals concurrently to the relay. Then, in the second phase, the relay broadcasts the signal to the terminals [7], [8]. For instance, the authors in [7] proposed a useful framework to solve the optimal power allocation problem for TWR network. Their work shows the advantages and the performance of optimal resource allocation over a fixed or discrete resource allocation. In [8], the work deals with joint single relay selection TWR-CR network using Amplify-and-Forward (AF) protocol in the high Signal-to-Noise (SNR) regime. However, to the best knowledge of the authors, the multiple relay selection problem in TWR-CR networks has not been discussed so far as it is the case for OWR-CR networks.

In this study, a multiple relay selection scheme for TWR-CR network with AF protocol is investigated. In the AF mode, the relays amplify the received signal before broadcasting it. We assume that the system works in a half duplex mode and channel reciprocity is considered. The main contributions of this paper can be summarized as follows:

• Formulate an optimization problem to maximize the sum rate of a TWR secondary network with AF protocol by taking into account the power budget of the system and the interference level tolerated by the PU.
• Design a practical low complexity heuristic approach to solve the formulated optimization problem.
• Compare the performance of the proposed algorithm with the performance of the optimal and Exhaustive Search (ES) solution in addition to the performance of the single relay selection scheme.

In our heuristic approach, we assume that each cognitive relay can operate with one of the available power levels (i.e. from zero to the maximum peak power) instead of the ON-OFF modes only (i.e. the relay either cooperates with its maximum power or does not cooperate at all) and this will contribute to the maximization of the rate by offering more degrees of freedom to the system.

The rest of this paper is organized as follows. Section II
II. SYSTEM MODEL

We consider a cognitive system consisting of one PU and a SN. The SN is constituted of two cognitive transceiver terminals \(T_1\) and \(T_2\) and \(L\) single antenna cognitive relays. A Non-Line of Sight (NLOS) link between \(T_1\) and \(T_2\) is considered. During the first time slot, known as the Multiple Access Channel (MAC) phase, \(T_1\) and \(T_2\) transmit their signals to the relays simultaneously, with a power denoted \(P_1\) and \(P_2\), respectively. In the second time slot, known also as the Broadcast Channel (BC) phase, the selected relays transmit the amplified signal to the terminals, with a power denoted \(P_{r_i}\), where \(i = 1, ..., L\). Half duplex channel case is considered as illustrated in Fig. 1. In this work, we assume that the PU and SU’s utilize the spectrum at the same time. In order to protect the PU, the received interference power due to the secondary nodes should be below a specific interference threshold denoted \(I_{th}\). Without loss of generality, all the noise variances are assumed to be equal to \(\sigma_n^2\).

Fig. 1. System model of the cooperative TWR-CR system.

Let us define \(\bar{P}, \bar{P}_r, h_{1r}, h_{2r}, h_{r}, h_{1p}, h_{2p}\), as peak power at the transceiver terminals, peak power at each relay, the channel gain between \(T_1\) and the \(i^{th}\) relay, the channel gain between \(T_2\) and the \(i^{th}\) relay, the channel gain between the \(i^{th}\) relay and the PU, the channel gain between \(T_1\) and the PU, and the channel gain between \(T_2\) and the PU, respectively. All the channel gains adopted in our framework are assumed to be Rayleigh fading channel gains and constant during the two transmission time slots. Furthermore, full Channel State Information (CSI) is considered. We denote by \(x_1\) and \(x_2\) the signals transmitted by \(T_1\) and \(T_2\), respectively. It is assumed that \(\mathbb{E}(|x_1|^2) = \mathbb{E}(|x_2|^2) = 1\), where \(\mathbb{E}(\cdot)\) denotes the expectation operator.

III. MULTIPLE RELAY SELECTION AND PROBLEM FORMULATION

For simplicity and without loss of generality, we assume that \(P_1 = P_2 = P\), where \(P\) is the transmitted power allocated for the cognitive transceivers. In the first time slot, the received signal at the \(i^{th}\) relay is given by

\[
y_{r_i} = \sqrt{P}h_{1r_i}x_1 + \sqrt{P}h_{2r_i}x_2 + n_{r_i},
\]

where \(n_{r_i}\) is the additive Gaussian noise at the \(i^{th}\) relay.

During the second time slot, each active relay amplifies \(y_{r_i}\) by multiplying it by \(w_i\) and broadcasts it to the terminals \(T_1\) and \(T_2\). The received signals in the BC phase are given by

\[
y_1 = \sum_{i=1}^{L} \epsilon_i w_i \left( \sqrt{P}(h_{1r_i}h_{1r_i}x_1 + h_{2r_i}h_{2r_i}x_2) + h_{1r_i}n_i \right) + n_1,
\]

\[
y_2 = \sum_{i=1}^{L} \epsilon_i w_i \left( \sqrt{P}(h_{2r_i}h_{1r_i}x_1 + h_{2r_i}h_{2r_i}x_2) + h_{2r_i}n_i \right) + n_2,
\]

where \(n_1\) and \(n_2\) are the additive Gaussian noise at \(T_1\) and \(T_2\), respectively. In (2), \(\epsilon_i\) is a binary variable denoting whether the \(i^{th}\) relay is active or not and it is given by

\[
\epsilon_i = \begin{cases} 1, & \text{if the } i^{th} \text{ relay is selected.} \\ 0, & \text{otherwise.} \end{cases}
\]

By using the knowledge of the CSI and channel reciprocity, the terminals can remove the self interference by eliminating their own signal (i.e., \(x_1\) for \(T_1\) and \(x_2\) for \(T_2\)). After the self interference cancelation, the SNR at \(T_1\) and \(T_2\) are given by

\[
\gamma_1 = \frac{P \left( \sum_{i=1}^{L} \epsilon_i |w_i h_{1r_i} h_{2r_i}| \right)^2}{\sigma_n^2 \left( 1 + \sum_{i=1}^{L} \epsilon_i |w_i h_{1r_i}|^2 \right)^2},
\]

\[
\gamma_2 = \frac{P \left( \sum_{i=1}^{L} \epsilon_i |w_i h_{1r_i} h_{2r_i}| \right)^2}{\sigma_n^2 \left( 1 + \sum_{i=1}^{L} \epsilon_i |w_i h_{2r_i}|^2 \right)^2},
\]

respectively. The relay power of the \(i^{th}\) relay node can be expressed as

\[
P_{r_i} = \mathbb{E}(|w_i y_{r_i}|^2) = (P|h_{1r_i}|^2 + P|h_{2r_i}|^2 + \sigma_n^2) |w_i|^2.
\]

From equation (5), the value of \(|w_i|^2\) can be expressed as

\[
|w_i|^2 = \frac{\sqrt{P_{r_i}}}{\sqrt{P|h_{1r_i}|^2 + P|h_{2r_i}|^2 + \sigma_n^2}}.
\]

By substituting the value of \(|w_i|^2\) into (4), the SNRs become
optimal power at the terminals MAC phase, the power allocation of both terminals depends in Section III, we solve it time slot per time slot. During the constraint in the first time slot, and in the second time slot, while the constraints (12) and (13) represent the interference constraints (10) and (11) represent the peak power constraint the state and the transmit power of each relay, respectively. The CR multiple relay selection can now be formulated as

$$I_r = \max \{ P_r, L \} \min \{ \sum_{i=1}^{L} P_{hi, r} | h_{1, r} |^2, \sum_{i=1}^{L} P_{hi, r} | h_{2, r} |^2 \}$$

Thus, the sum rate of the TWR can be written as

$$R = \frac{1}{2} \log_2 (1 + \gamma_1) + \frac{1}{2} \log_2 (1 + \gamma_2).$$

The sum rate maximization optimization problem of TWR-CR multiple relay selection can now be formulated as

$$\max_{P_r, \epsilon} R \quad \text{s.t.} \quad 0 \leq P \leq \bar{P},$$

$$0 \leq P_r, \forall i = 1, ..., L,$$

$$P(|h_{1, r}|^2 + |h_{2, r}|^2) \leq I_{th},$$

$$\sum_{i=1}^{L} \epsilon_i P_{r, i} \leq I_{th},$$

$$\epsilon_i \in \{0, 1\}, \forall i = 1, ..., L,$$

where $\epsilon = [\epsilon_1, ..., \epsilon_L]$ and $P_r = [P_{r, 1}, ..., P_{r, L}]$ are the decision variables of our formulated optimization problem that contain the state and the transmit power of each relay, respectively. The constraints (10) and (11) represent the peak power constraint at the terminals, and at each cognitive relay, respectively, while the constraints (12) and (13) represent the interference constraint in the first time slot, and in the second time slot, respectively.

IV. PROPOSED ALGORITHM

In order to simplify the formulated optimization problem in Section III, we solve it time slot per time slot. During the MAC phase, the power allocation of both terminals depends essentially on two constraints: the peak power constraint (10) and the interference constraint (12). For this reason, the optimal power at the terminals $P^*$ can be expressed as

$$P^* = \min \left( \frac{I_{th}}{|h_{1, r}|^2 + |h_{2, r}|^2}, \bar{P} \right).$$

Indeed, if the power at the terminals $\bar{P}$ affects the performance of the PU, then the power is reduced to $\frac{I_{th}}{|h_{1, r}|^2 + |h_{2, r}|^2}$. In the BC phase, we need to find the optimal power allocation over relays (i.e. $P^*_r$) in order to maximize the sum rate of SN without affecting the Quality of Service (QoS) of PU measured by $I_{th}$. The optimization problem during the second time slot is therefore given by

$$\max_{P_r, \epsilon} R \quad \text{s.t.} \quad (11), (13), (14).$$

The optimal solution for our non linear optimization problem formulated in (16) is difficult to find due the existence of binary variables $\epsilon_i, i = 1, ..., L$ [9]. Therefore, we deal with heuristic approaches to find suboptimal solutions to the problem.

In this section, we propose an iterative quantization algorithm with discrete number of power levels from zero to the peak relay power. In fact, each relay can transmit the amplified signal using one of the power level between 0 and $\bar{P}$. Thus, the state and the transmit power of each relay, respectively. The number of quantization levels. In this way, cognitive relays have more flexibility to allocate their powers in the case where continuous power distribution is not available. This method is considered as a generalization of the ON-OFF modes where relays can either transmit or keep silent.

**Algorithm 1 Proposed Algorithm**

- **Input:** $N, I_{th}, \sigma_n^2, \bar{P}, P_r, L, h_{1, r}, h_{2, r}, h_{r, p}, h_{1, p}$, and $h_{2, p}$.

- $P^* = \min \left( \frac{I_{th}}{|h_{1, r}|^2 + |h_{2, r}|^2}, \bar{P} \right)$.

- **Initialization:** $R_{max} = 0, P_r = \bar{P}, \epsilon = [0, ..., 0], L^V_{opt} = \emptyset$.

while $P_r = 0$

- $l = 1$.

while $l \leq L$ and $l \notin L^V_{opt}$

- $\epsilon^{int} = \epsilon$

- $\epsilon^{int}(l) = 1$.

- $R^{(l)} = \text{Compute Rate}(I_{th}, \sigma_n^2, P^*, P_r, L, h_{1, r}, h_{2, r}, h_{r, p})$.

- $l = l + 1$

end while

- Find $l_{opt} s.t. R_{opt} = \max_l R^{(l)}$.

if $R_{opt} > R_{max}$ then

- $\epsilon(l_{opt}) = 1$.

- $R_{max} = R_{opt}$.

- $L^V_{opt} = L^V_{opt} \cup \{l_{opt}\}$.

else

- $P_r = P_r - \frac{P_r}{N - 1}$.

end if

end while

Compute Rate function

if constraint (13) is satisfied then

- Compute the sum rate using equation (8).

else

- $R^{(l)} = 0$.

end if

A. Iteration-Quantization Algorithm

We assume that each relay has $N$ power levels from zero to the maximum power, i.e., a relay cooperates with one of the quantized power in $S$ without interfering with the PU. In the proposed algorithm, we aim to maximize the sum rate by transmitting the signals with the maximum number of relays powered with the maximum possible power without affecting the PU QoS. At the beginning, the transmit powers of all relays...
are fixed to $\bar{P}_r$ (i.e. the highest power level in the discrete quantization set $S$). The algorithm selects the relay that offers the highest $R$ and satisfies the constraint (13) at the same time. Then, it tries to add the maximum number of relays that can contribute in maximizing the sum rate. If, during this process, constraint (13) is not satisfied, then the new active relays have to be powered with the next lower power existing in the discrete quantized power set ($P_{r_i} \in S$). At the end, the algorithm converges when $P_{r_i}$ reaches 0 (i.e. no more relay can be selected even with the lowest non-zero power). The proposed algorithm is given in Algorithm 1.

B. Complexity Analysis

The formulated problem in (16) can be of course solved via an ES by investigating all possible combinations. This depends on $L$ (i.e. the number of relays in SN) and $N$ (i.e. the number of quantization level). Therefore, the ES algorithm requires $\sum_{i=0}^{L} (\begin{pmatrix} N \vphantom{\bar{P}_r} \\ I \end{pmatrix}) (N - 1)^i = O(N^L)$ operations to find the solution [10]. However, our proposed algorithm requires $(N - 1)L^2$ operations to reach a suboptimal solution. It is worth to notice that, the ES algorithm is not a practical choice due to its high complexity especially for a large number of relays $L$ and a high quantization level $N$. Hence, our proposed algorithm is able to reach a suboptimal solution with a considerable saving in terms of computational complexity. In addition to that, simulation results in Section V show that our proposed algorithm achieves almost the same performance as the ES method.

V. SIMULATION RESULTS

In this section, simulation results are presented to show the performance of the proposed algorithm for multiple relay selection TWR-CR network. The variance $\sigma^2_n$ is assumed to be equal to $10^{-4}$. Also, we assume that all cognitive elements have the same peak power, i.e., $P_{r_i} = \bar{P}_r$ and that all channels are assumed to be independent and identically distributed (i.i.d) Rayleigh fading channels. The simulations are performed under the scenario given in Fig.1.

Fig.2 shows a comparison between the performance of the proposed algorithm and the optimal solution with continuous power distributions. We plot the achieved secondary sum rate versus $\bar{P}_r$ for different values of $I_{th} = \{10, 20\}$ dBm and different values of $L = \{6, 10\}$. We can notice, in low SNR region, the proposed algorithm and the optimal solution have almost the same sum rate, while in the high SNR region, a gap between both methods is obtained. This gap is increasing with higher $P_{r_i}$ values. This is justified by the fact that starting from a certain value of $\bar{P}_r$, the system can not supply the relays with the whole power budget. Hence, more relays are deactivated. In fact, because of the use of discrete power distributions and with high values of $\bar{P}_r$, the constraint (13) can be affected. For this reason, we introduce the discretization set to get more degrees of freedom by increasing $N$ and as such enhance the SN sum rate. For instance, for $L = 6$ and $I_{th} = 20$ dBm, we were able to improve the achievable data rate by more than 150% going from 3 bits/s/Hz to more than 7.5 bits/s/Hz by having $N = 6$ instead of $N = 2$ (i.e. ON-OFF mode) when $\bar{P}_r = 30$ dBm. It should be noted that with the proposed algorithm, when $N \rightarrow \infty$, we achieve the performance of the optimal solution.

The performances of the ES algorithm, proposed algorithm, and the single relay selection with discrete $P_{r_i}$, are depicted in Fig.3. It is worth to mention that we can achieve higher cognitive sum rate by increasing the relay power budget for a fixed interference threshold up to a certain level. This can be justified by the fact that increasing the relay power budget will amplify the interference power due unlicensed users. For instance, Fig.3(a) and Fig.3(b) plot the cognitive sum rate and the average number of active relays versus the peak relay power for $L = 6$ and $N = 2$. It is shown that the proposed algorithm achieves almost the same SN sum rate of the ES algorithm by powering almost the same number of relays. However, by increasing $N$, we notice a degradation

![Fig. 2. Achieved sum rate versus the peak power $\bar{P}_r$ for the optimal and the proposed algorithm for multiple relay selection with different values of $I_{th}$ and $N$ (a) $L = 6$, (b) $L = 10$.](image-url)
of around 0.5 bits/s/Hz of the proposed algorithm at the SN sum rate peak comparing to the ES method while the same performance is reached otherwise as shown in Fig.3(c) - Fig.3(d). Nevertheless, the performance of our approach is less affected at the SN sum rate peak if we increase the number of relays \( L \) as it is shown in Fig.3(e) and Fig.3(f). Indeed, increasing \( L \) provides more possibilities than increasing \( N \) according to the proposed algorithm complexity \( (N-1)L^2 \), and thus a better suboptimal solution can be reached.

### TABLE I: COMPLEXITY COMPARISON

<table>
<thead>
<tr>
<th>( N, L )</th>
<th>ES Algorithm</th>
<th>Proposed Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 2, L = 6 )</td>
<td>64</td>
<td>36</td>
</tr>
<tr>
<td>( N = 2, L = 10 )</td>
<td>1024</td>
<td>100</td>
</tr>
<tr>
<td>( N = 6, L = 6 )</td>
<td>46656</td>
<td>180</td>
</tr>
<tr>
<td>( N = 2, L = 16 )</td>
<td>65536</td>
<td>256</td>
</tr>
<tr>
<td>( N = 6, L = 10 )</td>
<td>( 6 \times 10^2 )</td>
<td>500</td>
</tr>
<tr>
<td>( N = 500, L = 6 )</td>
<td>( 16 \times 10^{12} )</td>
<td>2994</td>
</tr>
<tr>
<td>( N = 500, L = 10 )</td>
<td>( 10 \times 10^{26} )</td>
<td>49900</td>
</tr>
</tbody>
</table>

In order to make a fair comparison with the single relay selection method, where the relay offering the highest \( R \) and that does not deteriorate the QoS of the PU is selected, we supply the selected relay with the same power used by all relays in our proposed algorithm. We notice that our algorithm outperforms the single relay selection scheme for all values of \( P_r, L, N, \) and \( I_{th} \). This proves that it is more beneficial for TWR-CR network to employ multiple relays instead of using a single relay.

### VI. CONCLUSION

In this paper, a practical low complexity heuristic approach is designed to maximize the SN achievable rate of a multiple relay selection scheme for two way relaying cognitive radio system with discrete power distributions. We have analyzed the performance of the proposed algorithm and compared it to the ES method and the optimal solution with continuous power distributions. The proposed algorithm is able to achieve almost the same performance as both schemes with a considerable saving in terms of computational complexity. In our ongoing task, we are working on deriving a closed form solution to the optimal scheme with continuous power distribution and applying our algorithm to OWR-CR networks.

### REFERENCES


